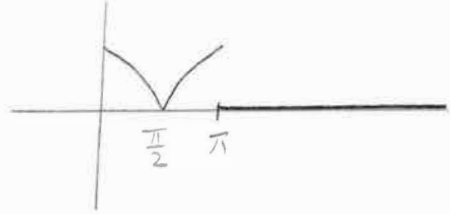
[2] $T=\pi$

$$\boxed{f_T(t)=\begin{cases}\cos t, & \text{IF } 0\le t<\frac{\pi}{2}\\ -\cos t, & \text{IF } \frac{\pi}{2}\le t<\pi \\ 0, & \text{IF } t\ge\pi\end{cases}}\quad (7)$$



ALL MARKED ITEMS (2) POINTS UNLESS OTHERWISE INDICATED

$$= \cos t + u(t-\tfrac{\pi}{2})(-\cos t-\cos t) + u(t-\pi)(0- -\cos t)$$

$$= \cos t - 2u(t-\tfrac{\pi}{2})\cos t + u(t-\pi)\cos t \quad (4)$$

$$\mathcal{L}\{f_T(t)\} = \frac{s}{s^2+1} - 2e^{-\frac{\pi s}{2}}\mathcal{L}\{\cos(t+\tfrac{\pi}{2})\} \quad (4)$$
$$+ e^{-\pi s}\mathcal{L}\{\cos(t+\pi)\} \quad (4)$$

$$= \frac{s}{s^2+1} - 2e^{-\frac{\pi s}{2}}\mathcal{L}\{-\sin t\} \; (3) + e^{-\pi s}\mathcal{L}\{-\cos t\}$$

$$= \frac{s}{s^2+1} + \frac{2e^{-\frac{\pi s}{2}}}{s^2+1} - \frac{se^{-\pi s}}{s^2+1} \quad (4) \qquad \mathcal{L}\{f\} = \frac{\mathcal{L}\{f_T\}}{1-e^{-\pi s}}$$

$$s^3Y - s^2\overset{2}{y(0)} - s\overset{-8}{y'(0)} - \overset{3}{y''(0)} \quad (4)$$
$$-3(s^2Y - s\overset{2}{y(0)} - \overset{-8}{y'(0)})$$
$$+20Y = \frac{s+2e^{-\frac{\pi s}{2}} - se^{-\pi s}}{(s^2+1)(1-e^{-\pi s})} \quad (5)$$

$$(s^3-3s^2+20)Y - 2s^2 + 8s - 3 + 6s - 24$$
$$= (s^3-3s^2+20)Y - 2s^2 + 14s - 27 = \frac{s+2e^{-\frac{\pi s}{2}} - se^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$$

$$\mathcal{L}\{y\} = \frac{2s^2-14s+27}{s^3-3s^2+20} + \frac{s+2e^{-\frac{\pi s}{2}} - se^{-\pi s}}{(s^2+1)(s^3-3s^2+20)(1-e^{-\pi s})} \quad (4)$$

[3] $\mathcal{L}^{-1}\left\{\frac{1}{(s-4)^6}\right\} = \frac{1}{5!}t^5 e^{4t} = \frac{1}{120}t^5 e^{4t}$ 4

$$\frac{1}{1+e^{-4s}} = 1 - e^{-4s} + e^{-8s} - e^{-12s} + e^{-16s} - \cdots = \sum_{n=0}^{\infty}(-1)^n e^{-4ns}$$

$$\frac{e^{-2s}}{1+e^{-4s}} = e^{-2s} - e^{-6s} + e^{-10s} - e^{-14s} + e^{-18s} - \cdots = \sum_{n=0}^{\infty}(-1)^n e^{-(4n+2)s}$$

$$\frac{e^{-4s}}{1+e^{-4s}} = e^{-4s} - e^{-8s} + e^{-12s} - e^{-16s} + e^{-20s} - \cdots = \sum_{n=0}^{\infty}(-1)^n e^{-(4n+4)s}$$

$$= \sum_{n=1}^{\infty}(-1)^{n-1} e^{-4ns}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-4)^6}\sum_{n=0}^{\infty}(-1)^n e^{-(4n+2)s} - \frac{1}{(s-4)^6}\sum_{n=1}^{\infty}(-1)^{n-1} e^{-4ns}\right\}$$

$$= \sum_{n=0}^{\infty}(-1)^n \mathcal{L}^{-1}\left\{\frac{1}{(s-4)^6}e^{-(4n+2)s}\right\} - \sum_{n=1}^{\infty}(-1)^{n-1}\mathcal{L}^{-1}\left\{\frac{1}{(s-4)^6}e^{-4ns}\right\}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{120}u(t-4n-2)(t-4n-2)^5 e^{4(t-4n-2)}$$ 4

$$- \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{120}u(t-4n)(t-4n)^5 e^{4(t-4n)}$$ 4

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-4)^6}e^{-ks}\right\} = u(t-k)\cdot\frac{1}{120}(t-k)^5 e^{4(t-k)}$$ 5

[4] $\boxed{f(t) = \begin{cases} 600t, & \text{IF } 0 \le t < 2 \\ -400t + 2000, & \text{IF } 2 \le t < 5 \\ 0, & \text{IF } t \ge 5 \end{cases}}$ 7

$$= 600t + u(t-2)(-400t + 2000 - 600t) + u(t-5)(0 - (-400t + 2000))$$

$$= \boxed{600t + u(t-2)(2000 - 1000t) + u(t-5)(400t - 2000)} \quad 4$$

$$\mathcal{L}\{f(t)\} = \frac{600}{s^2} + e^{-2s}\mathcal{L}\{2000 - 1000(t+2)\} \quad 4$$
$$+ e^{-5s}\mathcal{L}\{400(t+5) - 2000\} \quad 4$$

$$= \frac{600}{s^2} + e^{-2s}\mathcal{L}\{-1000t\} \; 1 + e^{-5s}\mathcal{L}\{400t\} \; 1$$

$$= \frac{600}{s^2} - \frac{1000e^{-2s}}{s^2} + \frac{400e^{-5s}}{s^2} \quad 4$$

$$s^2Y - s\underbrace{y(0)}_{-6} - \underbrace{y'(0)}_{20} + 8(sY - \underbrace{y(0)}_{-6}) + 20Y = \frac{600}{s^2} - \frac{1000e^{-2s}}{s^2} + \frac{400e^{-5s}}{s^2}$$

$$(s^2+8s+20)Y + 6s - 20 + 48$$

$$= (s^2+8s+20)Y + 6s + 28 = \frac{600}{s^2} - \frac{1000e^{-2s}}{s^2} + \frac{400e^{-5s}}{s^2}$$

$$Y = -\frac{6s+28}{(s+4)^2+4} + \frac{600}{s^2((s+4)^2+4)} - \frac{1000e^{-2s}}{s^2((s+4)^2+4)} + \frac{400e^{-5s}}{s^2((s+4)^2+4)}$$

4

$$\frac{6s+28}{(s+4)^2+4} = \frac{6(s+4)+4}{(s+4)^2+4} = \frac{6(s+4)+2(2)}{(s+4)^2+4}$$

$$\mathcal{L}^{-1}\left\{-\frac{6s+28}{(s+4)^2+4}\right\} = -6e^{-4t}\cos 2t - 2e^{-4t}\sin 2t \quad 4$$

$$\frac{200}{s^2((s+4)^2+4)} = \frac{\overset{-4}{\cancel{A}}}{s} + \frac{\overset{10}{\cancel{B}}}{s^2} + \frac{\overset{4}{\cancel{C}}(s+4) + \overset{3}{\cancel{D}}(2)}{(s+4)^2+4}$$ 4

$$200 = As((s+4)^2+4) + B((s+4)^2+4) + C(s+4)s^2 + D(2s^2)$$

$s=0$: $200 = B(20) \longrightarrow B = 10$

$s=-4$: $200 = A(-4)(4) + B(4) + D(32) = -16A + 40 + 32D$

5 $160 = -16A + 32D \longrightarrow 10 = -A + 2D \longrightarrow D = \frac{1}{2}(A+10)$

COEF s^3: $0 = A + C \longrightarrow C = -A$

COEF s: 4 $0 = 20A + 8B \longrightarrow A = -\frac{2}{5}B = -4$

$C = 4$

$D = 3$

SANITY CHECK: $s = -2$

$$\frac{200}{4(8)} = \frac{50}{8}$$

$$\boxed{\frac{-4}{-2} + \frac{10}{4} + \frac{4(2)+3(2)}{8} = \frac{16+20+14}{8} = \frac{50}{8}}$$

$$\mathcal{L}^{-1}\left\{\frac{-4}{s} + \frac{10}{s^2} + \frac{4(s+4)+3(2)}{(s+4)^2+4}\right\}$$

$$= -4 + 10t + 4e^{-4t}\cos 2t + 3e^{-4t}\sin 2t$$ 7

$$y = -6e^{-4t}\cos 2t - 2e^{-4t}\sin 2t$$
$$+ 3(4e^{-4t}\cos 2t + 3e^{-4t}\sin 2t + 10t - 4)$$
$$+ u(t-2)(-5(4e^{-4(t-2)}\cos 2(t-2) + 3e^{-4(t-2)}\sin 2(t-2) + 10(t-2) - 4)$$ 7
$$+ u(t-5)(2(4e^{-4(t-5)}\cos 2(t-5) + 3e^{-4(t-5)}\sin 2(t-5)) + 10(t-5) - 4)$$ 7

$$
\begin{aligned}
y = {} & 6e^{-4t}\cos 2t + 7e^{-4t}\sin 2t + 30t - 12 \\
& + u(t-2)\left(-20e^{-4t+8}\cos(2t-4) - 15e^{-4t+8}\sin(2t-4) - 50t + 120\right) \\
& + u(t-5)\left(8e^{-4t+20}\cos(2t-10) + 6e^{-4t+20}\sin(2t-10) + 20t - 108\right)
\end{aligned}
$$

$$
= \begin{cases}
6e^{-4t}\cos 2t + 7e^{-4t}\sin 2t + 30t - 12, & \text{IF } 0 \le t < 2 \\[2ex]
\begin{aligned} & 6e^{-4t}\cos 2t + 7e^{-4t}\sin 2t \\ & - 20e^{-4t+8}\cos(2t-4) - 15e^{-4t+8}\sin(2t-4) \\ & - 20t + 108, \end{aligned} & \text{IF } 2 \le t < 5 \quad 5 \\[2ex]
\begin{aligned} & 6e^{-4t}\cos 2t + 7e^{-4t}\sin 2t \\ & - 20e^{-4t+8}\cos(2t-4) - 15e^{-4t+8}\sin(2t-4) \\ & + 8e^{-4t+20}\cos(2t-10) + 6e^{-4t+20}\sin(2t-10), \end{aligned} & \text{IF } t \ge 5 \quad 5
\end{cases}
$$